Analog Computation in Computational Cognitive Neuroscience

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May 30, 2018

1 Abstract

Computation plays a unique explanatory role in cognitive science and neuroscience: what brains do is explained in terms of the computations they perform. What precisely that means remains unclear, but it is clear that brains do not compute in the same way that standard digital computers do. Very little of theoretical computer science illuminates how the brain computes. However, it does seem that brains might perform analog computation. Unfortunately, analog computation is not well-understood, and is often thought to simply mean computation over continuous variables. While that is often true, it is not the essential feature of analog computation, as I argue here. Instead, analog computation is the processing of analog representations, which are representations that physically co-vary monotonically with what they represent. Making clear what analog computation really is helps to make clear the sense in which brains compute, whether they do so continuously or discretely.

Keywords: computation; analog; digital

2 Computational Explanation

Relative to most other sciences, computation plays a unique explanatory role in the cognitive sciences.[1] Virtually all sciences use computation as a way of constructing models of various phenomena. Computational geosciences, computational astrophysics, and computational economics all use computer simulation and modeling techniques to explain various phenomena. The cognitive sciences use computation in this way as well, but there is another, perhaps more interesting way in which the notion of computation is deployed in these cases. Specifically, cognitive scientists explain the cognitive capacities and sub-capacities of the mind and brain in terms of the computations that certain systems perform (Piccinini, 2006). It is not merely that minds and brains can be simulated via computational methods, but that they literally perform computations themselves. Thus, while a computational astrophysicist might simulate the evolution of stellar dynamics with a complex computational system, no astrophysicist claims that a star does what it does because stars are literally computing something. But cognitive scientists claim just that: brains do what they do because they literally perform certain computations.

Unfortunately, what precisely it means to say that the brain performs computations is not completely clear. For some, this means that the brain encodes and processes information (Koch, 1999), for others, this means the brain traffics in representations (London & Häusser, 2005); while this may seem to amount to the same thing, there are good reasons to think of computation and information processing as distinct (Piccinini & Scarantino, 2011). Whatever the answer is to this question, it has become clear that the sense in which the mind/brain computes must be different from digital computation of the kind studied in computer science. In the past, some have argued that the all-or-none behavior of the action potential suggests an analogy to the binary nature of digital computation (von Neumann, 1958); it is now clear that the similarities between neural firing and digital computation do not meaningfully support the view that the brain is a computer. Thus, if the brain is a computer, it must be some other kind.

3 Analog Computation

The vast majority of discussions on computation take digital computation to be the beginning and the end of the story. Turing's work on computable functions provides us with the theoretical underpinnings of computability theory by way of the Turing Machine and other automata or formal frameworks, and the subsequent development of physical realizations of these automata has given us the incredibly powerful digital computers with which we are all familiar.

What's missing from this view of computation is analog computation. To many, this simply means computation involving continuous or real-valued quantities, but that does a disservice to the realities of analog compu-

[1] By "cognitive sciences," I mean to include so-called traditional cognitive science, computational neuroscience, and cognitive neuroscience, among others.

tation. In fact, a close look at how actual analog computers worked shows that analog computers often used discontinuous quantities in interesting ways. For example, some problems of interest might need to be approximated by a piecewise-linear function when their mathematical characterization was unknown. One solution is to use what were called arbitrary function generators. A textbook introduces these components as follows:

> Such behavior presents almost insurmountable obstacles to purely mathematical investigation, but poses no particular difficulty to analog-computer investigation. Again, we are not solving equations, we are modeling systems. Thus if we can describe the input-output relationship…, all we need to do is provide an element on the computer which has the same relationship between its input and output voltages. Such elements are known as arbitrary function generators. (Peterson, 1967, p. 109)

These components were able to produce piecewise-linear approximations to any given function (hence arbitrary function generators). One example is given in Figure 1. The striking thing about this example is that the analog computer would be using a discontinuous function to approximate a continuous function of interest. Given the received view of analog computers as only involving continuous variables, this would seem to be nearly oxymoronic. And yet, this is how analog computers could work.

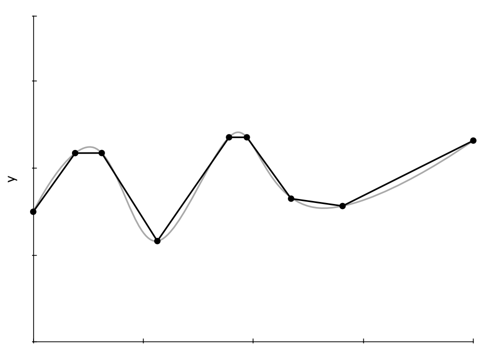

Figure 1: Continuous function (grey) modeled on an analog computer by an arbitrary function generator (black).

Another example of a discontinuous representation in analog computers is the representation of the step function. Certain phenomena exhibit discontinuous jumps between different levels. One way to model these jumps is via a continuous approximation. For example, an ideal square wave can be modeled as the sum of a number of sine (or cosine) waves. In Figure 2, a step function is shown with increasingly-accurate approximations, which are themselves sums of increasingly-many sine functions.

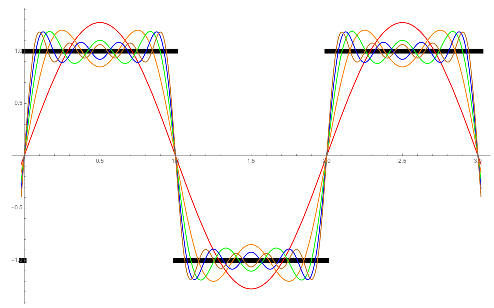

Figure 2: Step function (thick black line) approximated by successive sine approximations.

To represent such a function on an analog computer, one could use a similar continuous approximation method. However, a better method for representing a discontinuous function would be to use discontinuous functions. That is exactly what we find in analog computers. By using relays or switches, analog computers could directly represent step functions (for example), overcoming the limitations of using continuous approximations. In a discussion of the use of relays to incorporate a discontinuous voltage change in an analog computer, rather than using a continuous approximation (the diode limited amplifier circuit mentioned), the author of an analog computing monograph explains:

> For the problem being studied, it is not immediately obvious why the relay is needed. The voltage from the diode limited amplifier circuit can be made to closely approximate a delayed step function.…There are two reasons why this is not practical. The slope of the "step" function out of this circuit is not exactly zero after the original discontinuity. In the process of adding two of these step functions, a small error that increases with [time] would be applied to the integrator and cause an unwanted "drift" in the output of the integrator. (Ashley, 1963, p. 201)

These examples make it clear that continuity is not essential for analog computation. However, what is essential is analog representation in the sense of (Maley, 2011, 2018). On this view of analog representation, what makes a representation analog is that it varies monotonically with what it represents. Familiar examples are mercury thermometers and analog watches. These are analog not because they are fundamentally continuous, but because, in a literal, physical sense, they vary with what they represent. The hands of an analog watch rotate as time increases; the height of a mercury thermometer varies as temperature increases. Furthermore, an analog watch with hands that tick (i.e. vary in discrete steps) and a mercury thermometer that only increases in 1 degree increments are both still analog, even when they do not vary continuously.

4 Analog Neural Computation

Understanding what is essential about analog computation helps to make clear how brains might legitimately still be computers, even though they are not digital computers. For example, inter-neural phenomena such as firing rates are analog representations, as well as intra-neural phenomena such as electrical signaling in gap junctions and the analog modulation of individual neural spikes (Maley, 2018). This is true whether or not the underlying medium is really continuous or not: what matters is how the medium co-varies with what it represents.

This may also help to make sense of higher-level neural phenomena. This is somewhat speculative, given how little is known about larger-scale brain function and how ensembles of neurons work together to represent higher-level phenomena. However, there are some clear examples, such grid cells that represent location (Hafting, Fyhn, Molden, Moser, & Moser, 2005) and retinotopic coordinate maps represent various visual phenomena (Wandell, Dumoulin, & Brewer, 2007).

Most importantly, getting clear about different kinds of computation helps to bridge the ways that computation happens in both cognitive and neural processes. It is important to have a unified framework of computation so that researchers from different areas of neuroscience can make sense of what it means for different neural phenomena to perform computations.

5 Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. 1754974.

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